

# EFFECTIVE LAGRANGIAN FOR BARYONS AND BARYON-MESON INTERACTION

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Abstract

Expansion of model Nambu-Jona-Lasinio is considered and on its basis an effective lagrangian for mesons, baryons and baryon - meson interaction has been obtained.

The successes of quantum chromodynamics (QCD) in the description of high-energy interactions of hadrons are well-known. However its use has met many difficulties in medium and especially in low-energy range. The main difficulty is that the chromodynamical coupling constant  $\alpha_c$  in this range becomes large, that is why the common perturbation theory becomes inapplicable. Therefore the description of low-energy processes and properties of elementary particles requires putting forward the other approximate methods and models. Some of them are related to QCD. In this article the method allowing to derive the reported phenomenological meson and baryon Lagrangians, successfully describing low-energy physics of hadrons on the basis of effective quark interaction has been given.

Here we shall not discuss the constructing of an effective quark Lagrangian on the basis of QCD Lagrangian being examined for 4-th quark Lagrangian in [1,2]. We shall consider effective quark Lagrangian that is an extension of Lagrangian offered in [3,4]:

$$L = \bar{q}(i\gamma_\mu\partial_\mu - m_q)q + \frac{G_1}{2}[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2] - \frac{G_2}{2}[(\bar{q}\gamma_\mu\lambda^a q)^2 + (\bar{q}\gamma_5\gamma_\mu\lambda^a q)^2] + \sum C_i(\bar{q}\Gamma_i q)^4, \quad (1)$$

where  $\Gamma_i$ -matrices,  $i=(\mu, a, \alpha)$ ,  $\mu$ - spatial index,  $a$ - flavour,  $\alpha$ - colour. We shall now show, how meson and baryon fields are introduced and how phenomenological meson and baryon Lagrangians are obtained. With generating functional  $W(\bar{\eta}, \eta)$  this procedure can be conducted in three steps:

$$W(\bar{\eta}, \eta) = \frac{1}{N} \int dq d\bar{q} \exp[L(q, \bar{q}) + \eta\bar{q} + \bar{\eta}q], \quad (2)$$

where

$$\begin{aligned} L(q, \bar{q}) = & \bar{q}(i\gamma_\mu\partial_\mu - m_q^0)q + \frac{G_1}{2}[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2] - \\ & \frac{G_2}{2}[(\bar{q}\gamma_\mu\lambda^a q)^2 + (\bar{q}\gamma_5\gamma_\mu\lambda^a q)^2] + \\ & \sum_{i,I} \frac{\sqrt{2}G_i C^2}{2} [g_I(\bar{q}q\bar{q}T)_k^j (\lambda^a)_j^i \Gamma_I(Tqqq)_i^k \bar{q}_s (\lambda^a)_t^s \Gamma_I q^t + \\ & h_I(\bar{q}q\bar{q}T)_k^j \Gamma_I(Tqqq)_j^i (\lambda^a)_i^k \bar{q}_s (\lambda^a)_t^s \Gamma_I q^t]. \end{aligned}$$

In formula (2)  $I = S, P, V, A$ , and  $\Gamma_I$ - correspond to  $I, i\gamma_5, \gamma_\mu, \gamma_\mu\gamma_5$ ,  $G_i$ - is equal to  $G_1$  for S- and P- variants and to  $G_2$  for V- and A- variants,

$$(Tqqq)_i^k = (R_{V;i;i_1 i_2 i_3}^k + R_{V;i;i_1 i_2 i_3}^k) q_{b_1}^{i_1} q_{b_2}^{i_2} q_{b_3}^{i_3} \varepsilon^{b_1 b_2 b_3},$$

and the explicit forms for  $R_T$  and  $R_V$  are given in [5]. We shall proceed from the Lagrangian (2) to Lagrangian containing meson and baryon fields in addition to quark fields:

$$\begin{aligned}
L' = & \bar{q}[i\gamma_\mu\partial_\mu - m_0 + S + i\gamma_5\Pi + \gamma_\mu V_\mu + \gamma_5\gamma_\mu A_\mu + \\
& \sqrt{2}G_1g_1\bar{B}\lambda^a B\lambda_a - \sqrt{2}G_1h_1\bar{B}B\lambda^a\lambda_a + \sqrt{2}G_1g_2\bar{B}\gamma_5\lambda^a B\gamma_5\lambda_a - \\
& \sqrt{2}G_1h_2\bar{B}\gamma_5 B\lambda^a\gamma_5\lambda_a + \sqrt{2}G_2g_3\bar{B}\gamma_\mu\lambda^a B\gamma_\mu\lambda_a - \\
& \sqrt{2}G_2h_3\bar{B}\gamma_\mu B\lambda^a\gamma_\mu\lambda_a + \sqrt{2}G_2g_4\bar{B}\gamma_5\gamma_\mu\lambda^a B\gamma_5\gamma_\mu\lambda_a - \\
& \sqrt{2}G_2h_4\bar{B}\gamma_5\gamma_\mu B\lambda^a\gamma_5\gamma_\mu\lambda_a]q - \\
& \frac{S_a^2 + \Pi_a^2}{2G_1} - \frac{V_a^{\mu 2} + A_a^{\mu 2}}{2G_2} + \frac{g_1}{\sqrt{2}}\bar{B}SB - \frac{h_1}{\sqrt{2}}\bar{B}BS + \frac{g_2}{\sqrt{2}}\bar{B}\gamma_5\Pi B - \\
& \frac{h_2}{\sqrt{2}}\bar{B}\gamma_5 B\Pi + \frac{g_3}{\sqrt{2}}\bar{B}\gamma_\mu V_\mu B - \frac{h_3}{\sqrt{2}}\bar{B}\gamma_\mu B V_\mu + \frac{g_4}{\sqrt{2}}\bar{B}\gamma_\mu\gamma_5 A_\mu B - \\
& \frac{h_4}{\sqrt{2}}\bar{B}\gamma_\mu\gamma_5 B A_\mu,
\end{aligned} \tag{3}$$

where B- octet of baryons,  $S, \Pi, V_\mu, A_\mu$ —scalar, pseudo-scalar, vector and axial mesons respectively.

Lagrangian for standard  $\sigma$ -model, which describes scalar and pseudo-scalar mesons interaction has been obtained from Lagrangian (3) in [4]. In addition to Lagrangian for this model we shall obtain an effective Lagrangian for baryons. For that we shall consider the divergent quark loops of four types [4] and the diagrams of the mass operator type for baryons. In these loops there are different divergencies for mesons and baryons. The sum of the meson diagrams with quark loops results in the expression:

$$\begin{aligned}
& Tr \{ [p^2 I_2 + 2(I_1 + M^2 I_2)] \times [(s - M)^2 + \Pi^2] - \\
& I_2 \left( [(s - M)^2 + \Pi^2]^2 - [(s - M), \Pi]_-^2 \right)
\end{aligned}$$

Here the first term is a kinetic term (p-momentum of a meson). This term completely defines renormalization of meson fields.  $I_1$  and  $I_2$ - divergent integrals, which explicit forms have been given in [3], and  $S = gS^R, \Pi = g\Pi^R$ , where  $g = (4I_2)^{-\frac{1}{2}}$ . Performing similar computations for baryons we obtain from Lagrangian (3):

$$\bar{u}(p) \{ -i\gamma_\mu p_\mu I_3 + I_4 \} u(p),$$

where

$$I_{3,4} = \frac{3i}{2(2\pi)^4 V} [A^2 K_{3,4}(m_p, m_u, m_u) + B^2 K_{3,4}(m_p, m_d, m_d) + \\ C^2 K_{3,4}(m_p, m_s, m_s) + D^2 K_{3,4}(m_n, m_u, m_d) + E^2 K_{3,4}(m_\Lambda, m_u, m_s) + \\ F^2 K_{3,4}(m_\Sigma, m_u, m_s) + G^2 K_{3,4}(m_\Sigma, m_d, m_s)],$$

$$K_3(m_M, m_1, m_2) = \int \frac{d^4 s d^4 t 8(i\gamma_\mu s_\mu - m_2)(2i\gamma_\nu t_\nu + m_1)}{[(p+t+s)^2 + M^2](s^2 + m_2^2)(t^2 + m_1^2)}, \\ K_4(m_M, m_1, m_2) = \int \frac{d^4 s d^4 t 4[2i\gamma_\mu(t_\mu - s_\mu) + M](-i\gamma_\nu s_\nu + m_2)(2i\gamma_\alpha t_\alpha + m_1)}{[(p+t+s)^2 + M^2](s^2 + m_2^2)(t^2 + m_1^2)}.$$

After going to the other fields  $\bar{P}(p) = \sqrt{I_3} \bar{u}(p)$ ,  $P(p) = \sqrt{I_3} u(p)$  we have:

$$L = \bar{P}(p) \left\{ -i\gamma_\mu p_\mu + \frac{I_4}{I_3} \right\} P(p).$$

Thus an effective Lagrangian for protons has been obtained. An effective Lagrangian for other baryons can be derived in the same manner.

## REFERENCES

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